

### Problem 3

For the following series, write formulas for the sequences  $a_n$ ,  $S_n$ , and  $R_n$ , and find the limits of the sequences as  $n \rightarrow \infty$  (if the limits exist).

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} \cdots$$

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#### Solution

$$a_n = \frac{(-1)^n}{2^n}$$

$$S_n = \sum_{i=0}^n \frac{(-1)^i}{2^i} = \sum_{i=0}^n \left(-\frac{1}{2}\right)^i = \frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{1 - \left(-\frac{1}{2}\right)} = \frac{2}{3} + \frac{(-2)^{-n}}{3}$$

$$S = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left[ \frac{2}{3} + \frac{(-2)^{-n}}{3} \right] = \frac{2}{3}$$

$$R_n = S - S_n = \frac{2}{3} - \left[ \frac{2}{3} + \frac{(-2)^{-n}}{3} \right] = -\frac{(-2)^{-n}}{3}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{(-1)^n}{2^n} = 0$$

$$\lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \left[ -\frac{(-2)^{-n}}{3} \right] = 0$$